Course Structure for M.A./ M. Sc. Mathematics

Semester I

Core Courses

Course Code	Course Title	Lec Hr	Lab Hr	Tut	SSHr	Credits
MMAT1C001T	Linear Algebra	4	0	1	8	4
MMAT1C002T	Real Analysis	4	0	1	8	4
MMAT1C003T	Topology	4	0	1	8	4

Open Elective

Course Code	Course Title	Lec Hr	Lab Hr	Tut	SSHr	Credits
MMAT1O004T	Ordinary Differential Equations with Applications	4	0	1	8	4
MMAT1O005T	Introduction to Python Programming	4	2	0	8	4
MMAT1O006T	Introduction to Set Theory	3	0	0	4	2
MMAT1O007T	Classical Mechanics	3	0	0	4	2

*The candidate has to earn a minimum of 20 credits during semester-I. Apart from the three compulsory core courses, the candidate has to earn 8 credits from the open elective courses.

Semester II

Core Courses

Course Code	Course Title	Lec Hr	Lab Hr	Tut	SSHr	Credits
MMAT1C008T	Abstract Algebra	4	0	1	8	4
MMAT1C009T	Complex Analysis	4	0	1	8	4
MMAT1C0010T	Measure and Integration	4	0	1	8	4

Open Elective

Course Code	Course Title	Lec Hr	Lab Hr	Tut	SSHr	Credits
MMAT1O0011T	Probability and Statistics	4	0	1	8	4
MMAT1O0012T	Partial Differential Equations and Integral Equations	4	0	1	8	4
MMAT1O0013T	Introduction to Latex	2	4	0	4	2
MMAT2O0014T	Calculation of Variations	2	0	1	4	2

*The candidate has to earn a minimum of 20 credits during semester-II. Apart from the three compulsory core courses, the candidate has to earn 8 credits from the open elective courses.

Semester III

Course Code	Course Title	Lec Hr	Lab Hr	Tut Hr	SSHr	Credits
MMAT2O0015T	Fields and Galois Theory	4	0	1	8	4
MMAT2O0016T	Functional Analysis	4	0	1	8	4
MMAT2O0017T	Differential Geometry of Curves and Surfaces	4	0	1	8	4
MMAT2O0018T	Numerical Analysis	4	0	1	8	4
MMAT2O0019T	Discrete Mathematics	4	0	1	8	4
MMAT2O0020T	Fuzzy Theory and its Applications	4	0	1	8	4
MMAT2O0021T	Advanced Complex Analysis	4	0	1	8	4
MMAT2O0022T	Harmonic Analysis	4	0	1	8	4
MMAT2O0023T	Homological Algebra	4	0	1	8	4
MMAT2O0024T	Advanced Measure Theory	4	0	1	8	4
MMAT2O0025T	Optimization Techniques	4	0	1	8	4
MMAT2O0026T	Minor Project	4	0	0	8	4

*The candidate has to earn a minimum of 20 credits during semester-III from the open elective courses.

Semester IV

Course Code	Course Title	Lec Hr	Lab Hr	Tut Hr	SSHr	Credits
MMAT2O0027T	Finite Fields and Coding Theory	4	0	1	8	4
MMAT2O0028T	Finite Elements Methods	4	0	1	8	4
MMAT2O0029T	Commutative Algebra	4	0	1	8	4
MMAT2O0030T	Algebric Topology	4	0	1	8	4
MMAT2O0031T	Cryptography	4	0	1	8	4
MMAT2O0032T	Operator Theory	4	0	1	8	4
MMAT2O0033T	Topological Vector Spaces	4	0	1	8	4
MMAT2O0034T	Fourier Analysis	4	0	1	8	4
MMAT2O0035T	Stochastic Processes	4	0	1	8	4
	Minor Project#	4	0	1	8	4

#Continued from Semester III.

*The candidate has to earn a minimum of 20 credits in Semester IV from the open elective courses.

*The candidate who opts Minor Project in Semester III has to continue it in Semester IV as well.

*The evaluation of the Minor Project started in Semester III shall be done in Semester IV.

*The candidate has to earn a minimum of 80 credits to complete the Master's degree in Mathematics.

Programme out comes (Pos)

1. Demonstrate in depth knowledge of mathematics, both in theory and application.
2. To analyse complex problems in Mathematics and propose solutions using research based knowledge.
3. To Know the various specialised areas of advanced mathematics and its applications
4. To know the use of computers both as an aid and as a tool to study problems in Mathematics.
5. To obtain the accurate solutions for the community oriented problems via various mathematical models.
6. To attain the ability to identify, formulate and solve changing problems in Mathematics.
7. To work individually or as a team member or leader in uniform and multidisciplinary settings.
8. To inculcate the knowledge of formulation and apply the mathematical concepts which are suitable for real life applications.

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Programme Specific Outcomes (PSOs)

1. To communicate concepts of Mathematics and its applications.
2. To acquire analytical and logical thinking through various mathematical tools and techniques.
3. To investigate real life problems and learn to solve them through formulating mathematical models.
4. To motivate and prepare the students for research in various areas of mathematics.
5. To provide quality education and strong foundation in different areas of mathematics.

Course code: MMAT1C001T **Course title:** Linear Algebra

Objective. The aim of this course is to teach the students how to solve the linear system of equations using Gauss elimination, introduce the notion of vector space and classify the finite dimensional vector spaces, matrix representaion of a linear transformation, characterstic polynomial of a linear operator, diaogonalization of a linear operator, Bilinear forms: Symmetric and Hermintian forms, Orthogonal matrices and rotations, Spectral theorem for normal operator, application of spectral theorem to classify conics, rational and Jordan canonocal forms.

Course Contents

Unit-1

- Matrix, Operations on Matrices, Special types of Matrices, Elementary Matrices, Vectors in $\mathbb{R}^n$ and $\mathbb{C}^n$
- Row reduction, Rank of a matrix, Solution of Matrix equation $AX = B$, Determinants, Cramer's Rule

Unit-2

- Vector space: Definition and examples, Subspaces, Quotient Spaces, Linear span, Linear independence and dependence, Basis and Dimension, Finite dimensional vector spaces, Existence of basis, Computations with a basis

Unit-3

- Linear Transformation, The Matrix of Linear Transformation, Rank-nullity theorem, Effect of change of basis on Matrix of a linear transformation,
- Linear Operator, Eigenvalues and Eigenvectors, characteristic polynomial of a linear operator, Diagonalization

Unit-4

- Orthogonal matrices and rotations
- Bilinear forms: Symmetric forms, Hermitian forms, Orthogonality, Orthogonal Projection, Euclidean and Hermitian Spaces

Unit-5

- The Spectral theorem, Classification of conics
- Modules, Submodules, Structure theorem for finitely generated modules over a Principal ideal Domain (statement only), Rational and Jordan Canonical forms

Recommended Texts.

1. M Artin, Algebra, Second edition, PHI Learning Private Limited, New Delhi, 2012.
2. I N Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
3. K Hoffman and R Kunze, Linear Algebra, 2nd Edition, Prentice Hall, Englewood Cliffs, New Jersey, 1971.
4. S Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India, 2000.
5. S K Jain, A Gunawardena & P B Bhattacharya, Basic Linear Algebra with Matlab, Key College Publishing (Springer-Verlag) 2001.

Learning Outcome. After completing this course, the students will be able to check whether a given a square matrix is invertible and compute its inverse(if it exists), solve the linear system of equations, compute the inverse of a matrix, understand the properties of determinant function and its polynomial nature, to find all the bases of a vector space and linear transformations, compute the matrix of a linear transformation, compute the characteristic polynomial and eigenvalues of a linear operator, diagonalize a linear operator, understand the bilinear forms and geometries associated with them, diagonalize the normal operators, classify the conics, compute the rational and Jordan canonical forms.

Course code: MMAT1C002T **Course title:** Real Analysis

Objective. The aim of this course is to introduce the students to the Euclidean Space $\mathbb{R}^n$, properties of functions on $\mathbb{R}^n$, Reimann-Stieltjies intergral, sequences and series of functions, functions of bounded variation and properties of function of several variables.

Course Contents

Unit-1

- Euclidean Space $\mathbb{R}^n$, Open balls and open sets in $\mathbb{R}^n$, Structures of open sets in $\mathbb{R}^n$, Closed sets, Adherent and accumulation points, Closure of a set, Derived set
- Bolzano's Weierstrass theorem, Cantor Intersection theorem, Lindeloff covering theorem, Heine-Borel theorem, Compactness in $\mathbb{R}^n$

Unit-2

- Definition and existence of Reimann-Stieltjies intergral, conditions for R-S integrability, properties of the R-S intergral, Integration and differentiation.
- Fundamental theorem of calculus, Integration of vector valued functions, Rectifiable curves.

Unit-3

- Sequences and series of functions, Point-wise and uniform convergence, Cauchy's criterion for uniform convergence.
- Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity.

Unit-4

- Uniform convergence and Reimann-Stieltjies integration, uniform convergence and differentiation, Weierstrass approximation theorem
- Power series, uniqueness theorem for power series, Abel's and Tauber's theorems, Functions of bounded variation, continuous functions of bounded variation

Unit-5

- Function of several variables, linear transformation, Derivative of a real valued rule function, Directional derivatives, chain rule, Partial derivatives, and interchange of the order of the differentiation.
- Higher order derivatives, Taylor's theorem, Inverse function theorem, implicit function theorem, Jaçobians, Extremum problems with constraints, Legrange's multiplier method

Recommended Texts.

1. 1. T M Apostol, Mathematical Analysis, 2nd Edition, Narosa Publishing House, 2002.
2. 2. W Rudin, principles of Mathematical Analysis, 3rd Edition, McGraw-Hill International Editions, 1976.
3. 1. H L Royden amd P M Fitzpatrick Real Analysis, 4th Edition, PHI Learning Private Limited, 2004.
4. 2. D Somasundram and B Choudhary, A First Course in Mathematical Analysis, Corrected Edition, Narosa Publishing House, 2011.

Learning Outcome. Upon completing this course the students will be able to: Learn, apply and recognize topological properties of $\mathbb{R}^n$, properties of functions on $\mathbb{R}^n$, properties of Reimann-Stieltjies intergral, sequences and series of functions, functions of bounded variation and function of several variables.

Course code: MMAT1C003T **Course title:** Topology

Objective. This course aims to familiarize the students with the basic concepts of Topology. A preliminary knowledge of real analysis is essential.

Course Contents

Unit-1

- Topological Spaces: Definition and some examples, Metric spaces, Interior, Closure, and Boundary of a set, Basis and Sub-basis, First and second countable spaces, Continuous functions, Open and Closed Functions, Homeomorphism, Subspaces.

Unit-2

- Connected and disconnected Spaces, Results on Connectedness, Connected subsets of real Line, Applications of connectedness, Path Connected Spaces, Locally connected and Locally Path connected Spaces.

Unit-3

- Compact Spaces and Subspaces, Compactness and Continuity, Properties related to Compactness, One-Point Compactification, The Cantor Set.

Unit-4

- Finite Product spaces, Arbitrary products spaces, Tychnov's Theorem Comparison of Topologies, Quotient Spaces.
- Separation Axioms: T_0, T_1, and T_2 spaces.

Unit-5

- Regular Spaces, Normal Spaces, Separation by Continuous functions: Uryshon's Lemma, Completely regular spaces, Tietze extension theorem.

Recommended Texts.

1. F H Croom, Principles of Topology, Cengage Learning India Private Limited, New Delhi, First Indian Reprint 2008.
2. G F Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill.
3. James R Munkeres, Topology, Afirst course, Prentice Hall of Indi Pvt. Ltd., New Delhi, 2000.
4. J Dugundji, Topology, Allyn and Bacon, 1966 (reprinted in India by PHI Pvt. Ltd).
5. K D Joshi, Introduction to general Topology, Wiley, Eastern Ltd. 1983.
6. S T Hu, Elements of General Topology, Holden-Day, Inc. 1965.
7. W J Pervin, Foundations of General Topology, Academic Press Inc., New York, 1964.
8. S Willard, General Topology, Addison - Wesley, Reading, 1970.

Learning Outcome. Upon completing this course the students will be able to learn and recognize fundamental results of topological-spaces and are able to use and correlate these properties with the properties of $\mathbb{R}^n$.

Course code: MMAT1O005T **Course title:** Introduction to Phthon Programming

Objective. The aim of this course is teach the students the foundations Python programming language. By the end of course, the students will be able to write programs in ptthon using the most common structures. No provious exposure to programming is needed. The students will understand the benefits of programming; figure outhow the buildingblocks of programming fit together; and will be able to combine all of this knowledge to solve complex programming problems.

Course Contents

Unit-1 Introduction

- What is a programming language and what syntax means
- Benefits of the Python programming language
- Understand Functions
- Use basic functions and keywords to display data and perform arithmetic operations

Unit-2 Basics (Expressions and Variables)

- Data Types
- Expressions, numbers and type conversions
- Implicit vs Explicit conversions

Unit-3 Functions and Conditionals

- Defining functions and returning values using functions
- Compare values using equality operators and logical operators
- Build complex branching scripts utilizing if, else and elif statements

Unit-4 Loops (while and for)

- Implement while loops to continuously execute code
- Identify and fix infinite loops when using while loops
- Utilize for loops to iterate over sets of data
- Use the range() function to control for loops
- Identify and correct common errors when using loops

Unit-5 Strings Lists and Dictionaries

- Manipulate strings using indexing, slicing, and formatting
- Use lists and tuples to store, reference, and manipulate data
- Leverage dictionaries to store more complex data, reference data by keys, and manipulate data stored
- Combine these data types to construct complex data structures

Recommended Texts.

1. Mark Lutz, Learning Python: Powerful Object-Oriented Programming, 5th Edition
2. C. H. Swaroop, A Byte of Python

Course code: MMAT1O006T **Course title:** Introduction to Set Theory

Objective. This is a foundation course in Mathematics useful for everybody working in any areas of Mathematics. Pre-requisite for this course is under graduate elementary logic and set theory.

Course Contents

Unit-1

- The Axiom of choice and some of its equivalent forms: Motivation and historical remarks, family of sets and Cartesian product of family sets, partial ordered sets Hausdorff Maximality Principle , fixed point theorem (statement only), Zorn's lemma, Applications of Zorn's lemma, well ordering principle, equivalence of the above three concepts , Principle of transfinite induction.

Unit-2

- Denumerable and non- denumerable sets: finite and infinite sets, Equipotent of sets, examples and properties of denumerable and non -denumerable sets, cardinal numbers, ordering of the cardinal numbers, cardinal numbers of a power set, Cantor theorem, Schroder Berstein theorem (statement only), addition and multiplication of cardinal numbers, exponential of cardinal numbers, the continuum hypothesis and its generalization.

Unit-3

- Ordinal Numbers, Ordering of the ordinal numbers, addition and multiplication of ordinal numbers, set of ordinal numbers is well ordered, non existence of a set of all ordinals, problems and exercises based on these concepts.

Recommended Texts.

1. Shwu-Yeng T Lin, Set Theory with Applications, Mariner Pub. Co Enlarged 2nd Edition (1981)
2. Paul R Halmos, Naive set Theory, Springer- Verley New York Inc, 1974
3. Robert R. Stoll, Set Theory and Logic, W.H.Freeman and Co,1963

Learning Outcomes. After studying this course the student will be able to know about the Axiom of choice and some of its equivalent forms, Principle of transfinite induction, Cantor theorem, Schroder Berstein theorem, cardinal numbers and ordinal numbers.

Course code: MMAT1O007T **Course title:** Classical Mechanics

Objective. The goal of this paper is to introduce classical mechanics with an emphasis on fundamental understanding. The large-scale Lagrangian and Hamiltonian equations for dynamical systems are introduced. There is also a brief section on rigid body dynamics.

Course Contents

Unit-1

- Momentum and Kinetic energy, Motion about a fixed point, Euler's equation, General equation of motion for a single particle and system of N-number of particles, General solutions, Motion of heavy sphere in a cylinder and a cone, Motion under no-force, Torque, Poinst's representation of motion.

Unit-2

- Lagrange's equation of motion for holonomic systems, Velocity dependent potential, Conservation theorem and symmetric properties, Lagrange's multiplier for holonomic and non-holonomic systems, Lagrange's equation for impulsive motion.

Unit-3

- Hamiltonian of a dynamical system, Hamilton's canonical equation of motion, Cyclic coordinates The Routhian, Conservation of energy and momentum, Lagrange's method for small oscillation, Normal modes.

Recommended Texts.

1. Goldstein, H. Classical Mechanics, Second Edition, Narosa Publishing house, New Delhi, 2000.
2. Rana, N. C. and Joag, P. C. Classical Mechanics, (Tata-McGraw Hill, 1991).
3. Synge, J. L. and Griffith, B. A. Principles of Mechanics, McGraw-Hill, 1991

Learning Outcomes. After studying this course the student will be able to know about the Axiom of choice and some of its equivalent forms, Principle of transfinite induction, Cantor theorem, Schroder Berstein theorem, cardinal numbers and ordinal numbers.

Course code: MMAT1C008T **Course title:** Abstract Algebra

Objective. The aim of this course is to introduce the students to the basic notions of group theory and ring theory. In group theory, we focus on the structure of finite groups and classify all finite Abelian groups upto isomorphism. The ring theory part, our focus will be on the rings unique factorisation into ireducible elements and polynomial rings with unique factorisation.

Course Contents

Unit-1

- Groups: Definition and examples, Matrix and Permutation groups, subgroups, normal subgroups, Quotient group, group homomorphism, fundamental isomorphism theorems, Automorphism groups of $\mathbb{Z}_n$, $\mathbb{Z}$ and $\mathfrak{S}_3$, Examples and Exercises based on these topics
- Group action on a set, Orbit-Stablizer formula, Conjugation, Automorphism, Computation of automorphism groups of $\mathbb{Z}_n, \mathbb{Z}$ and $\mathfrak{S}_n$, examples and exercises based on these topics

Unit-2

- Class equation and its Applications, Cauchy theorem, examples and exercises based on these topics
- Sylow theorems for finite groups, Simple groups, Simplicity of the alternating group A_n; $n \geq 5$, examples and exercises based on these topics

Unit-3

- Direct sums, Structure theorem for finite Abelian groups and its applications, examples and exercises based on these topics
- Composition series, Jordan-Hölder theorem, Solvable groups, examples and exercises based on these topics

Unit-4

- Rings, subrings, Ideals, Quotient rings, ring homomorphism, Isomorphism theorems, Matrix and Polynomial rings, prime and maximal ideals, examples and exercises based on these topics
- Integral domain, Field of fractions of an integral domain, prime and irreducible elements, Unique factorization domain, examples and exercises based on these topics

Unit-5

- Principal ideal domains and Euclidean domains
- Polynomial rings over unique factorization domain: Gauss Lemma and Gauss theorem, Eisenstein criteria of irreducibility of polynomials, examples and exercises based on these topics

Recommended Texts.

1. N. Jacobson, Basis Algebra, Vol. I, second edition, Dover Publications, 2012.
2. I. N Herstein, Topics in Algebra, Wiley Eastern Ltd., Second Edition, New Delhi, 2011.
3. W. A. Adkins and S. H. Weintraub, Algebra An approach by module theory, Springer, 1999.
4. M. Artin, Algebra, Prentice-Hall of India, Second Edition, 2011.
5. N. S. Gopalakrishnan, University Algebra, New Age International(P) Ltd., Publishers, Second Edition: 1986, (Reprint: 2004)
6. I. S. Luther and I. B. S. Passi, Algebra, Vol 1-Groups, Vol II-Rings, Narosa Publishing House (Vol. I-1996, Vol. II-1999).

Learning Outcome. After completing this course, the students are expected to compute automorphism group of cyclic groups, characterize some finite groups and all finite abelian groups upto isomorphism, check the simplicity of some groups, compute the solvable and composition series for some groups, understand the relation between prime and irreducible elements in a ring, check the irreducibility of polynomials in $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$, construction of some finite fields.

Course code: MMAT1C009T **Course title:** Complex Analysis

Objective. The objective of this course is to introduce the fundamental ideas of the functions of complex variables and developing a clear understanding of the fundamental concepts of Complex Analysis such as analytic functions, complex integrals and a range of skills which will allow students to work effectively with the concepts.

Course Contents

Unit-1

- Review of complex numbers, Stereographic projection, Chordal distance, Multi-valued functions, Branches of multi-valued functions, with special reference to arg z, exponential functions, Logarithm function, power functions and phase factors. Analytic functions: Limit and continuity of complex functions, complex derivative, Singularities, Cauchy-Reinmann equations, Cauchy-Reinmann equations in polar form, Harmonic functions, Harmonic conjugate.

Unit-2

- Line integrals, Piecewise smooth path, Jordan curve, Green's theorem, Independence of path, Anti-derivative, fundamental theorem of calculus, Mean value property, Strict maximum principal (real and complex version), ML-estimate.

Unit-3

- Complex integration and analyticity: Cauchy's theorem, Cauchy Integral formula, Cauchy integral formulae for higher order derivatives.
- Liouville's theorem, Cauchy's inequality, Morera's theorem, Goursat's theorem, complex form of Cauchy-Riemann equations.

Unit-4

- Power series, radius of convergence, power series expansion of an analytic function: Taylor's expansion, Isolated singularities, Laurent Series. The residue calculus, Cauchy residue theorem, fractional residues, Jordan's lemna, Evaluation of integrals using residue theorem.

Unit-5

- Conformal mappings, Mobius transformations, composition of two Mobius transformations Translations, Dilations, Inversion, The Schwarz lemma, Conformal Self-maps of the unit disk, Mappings of the unit disk and upper half plane, The Riemann Mapping theorem (Statement only).

Recommended Texts.

1. TW Gamelin, Complex Analysis, Springer–Verlag, New York Berlin Heidelberg 2001.
2. Walter Rudin; Real and Complex Analysis, Tata Mc-Graw Hill,2006
3. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2005
4. J.W. Brown and R.V. Churchill, Complex variables and applications, Mc-Graw Hill International VIII-Edition, 2009
5. J.B. Conway, Function of One complex variable, Springer International Student Edition, 1980
6. L.V. Ahlfors, Complex Analysis, International Edition, McGraw Hill International Editions, 1979.

Learning Outcome. Upon completing this course the students will be able to: Learn and apply fundamental properties of complex analysis and are able to use these them in other areas of Mathematics, Physics and Engineering.

Course code: MMAT1C0010T **Course title:** Measure and Integration

Objective. The aim of this course is to study general theory of measure and integration. It is a pre-requisite course for Fourier Analysis and Wavelets and has lots of applications in functional analysis, Operator theory, integral equations, Probability theory and several branches of Physics.

Course Contents

Unit-1

- σ-algebra of sets, limits of sequences of sets, Generation of σ-algebras, Borel σ-algebras, Measure on a σ-algebra, Measurable spaces and measure spaces, Outer measures, construction of Measure by means of outer measure, Construction of outer measures by means of sequential covering class.

Unit-2

- Lebesgue measure on R, some properties of Lebesgue measure, Translation invariance of Lebesgue measure, Existence of non-Lebesgue measurable sets, Measurable functions, Operations with measurable functions. Equality almost everywhere, Sequence of measurable functions.

Unit-3

- Lebesgue Integration, Integration of step functions, Approximation theorem, Lebesgue integral of non-negative functions, Lebesgue integral of measurable functions.

Unit-4

- Convergence a.e., Almost uniform convergence, , Egoroff's Theorem, Convergence in measure, Convergence in mean, Cauchy sequence in measure.

Unit-5

- Fatous Lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem and applications.

Recommended Texts.

1. J. Yeh, Lectures on Real Analysis, World Scientific, 2000. Reference books:
2. M E Munroe, Measure and Integration, 2nd edition, Addison Wesley, 1971.
3. G De Barra, Measure theory and Integration, Wiley Eastern Ltd., 1987.
4. H L Royden, Real Analysis, 3rd edition, Macmillan, New York, 1988.

Learning Outcome. Upon completing this course the students will be able to: Learn fundamental concepts in Fourier Analysis, Wavelets, Functional analysis, Operator theory, Integral equations, Probability theory and several branches of Physics Physics and Engineering.

Course code: MMAT1C0011T **Course title:** Probability and Statistics

Objective. The goal of the course is to acquaint students with various probability distributions as well as to improve their abilities and understanding of sampling distributions and hypothesis testing.

Course Contents

Unit-1

- Review of probability theory including conditional probability, Some important theorems on probability, Exercise on probability and conditional probability, Independent Events, Total probability theorem and Bayes' Theorem, Addition and multiplication theorems of probability.

Unit-2

- Random Variables and Distribution function: discrete and continuous, Exercise on distribution functions, Two dimensional random variables: joint distribution function and marginal distribution, Expectation and moments about mean and origin, Covariance and conditional expectation and examples, Moment inequalities- Tchebyshef, Markov, Jensen, Moment generating function and characteristic function with their properties.

Unit-3

- Standard discrete probability distributions: Discrete uniform distribution, Bernoulli distribution, Binomial distribution, Poisson distribution, Geometric distribution, Negative binomial distribution with their properties and examples. Some important theorems based on these distributions.

Unit-4

- Standard continuous probability distributions: Continuous uniform distribution, Normal distribution, Exponential distribution, Gamma distribution or Erlang distribution, Weibull distribution, Triangular distribution, Standard Laplace (Double exponential) distribution, Cauchy distribution, with their properties and examples. Some important theorems based on these distributions.

Unit-5

- More on two dimensional random variables: probability and distributions and examples, Transformation of random variables with examples, Central limit theorem and its applications, Large sample theory: types, parameter and statistics, test of significance.

Recommended Texts.

1. S. C. Gupta and V. K. Kapoor: "Fundamentals of Mathematical Statistics", Sultan Chand and Sons, New Delhi.
2. S. Palaniammal, "Probability and Queueing Theory", PHI Learning Private Limited, Delhi.

Learning Outcome. Upon successful completion of this course, students will be:

1. Able to comprehend the essential ideas of probability, such as random variables, event probability, additive rules, and conditional probability.
2. Able to comprehend Bayes' theorem notion
3. Able to comprehend statistical ideas and measures at a basic level
4. Able to construct the central limit theorem notion
5. Understand Binomial, Geometrical, Negative Binomial, Pascal, Normal, and Exponential Distributions.
6. Be able to comprehend the ideas of various parameter estimation approaches, such as the method of moments, maximum likelihood estimation, and confidence intervals.
7. Possessing the ability to test hypotheses.

Course code: MMAT1C0012T **Course title:** Partial Differential Equations and Integral Equations

Objective. The main goal is to introduce the concepts of Partial Differential Equations (PDEs) and Integral Equations. Partial Differentials Equations act as mathematical models for many physical phenomena. The classical areas of Partial Differential Equations involve parabolic, hyperbolic and elliptic PDEs, Laplace, Heat and Wave equations. ature of Linear Integral Equations like Fredholm and Volterra type, eigen functions and resolvent kernel. It requires a basic knowledge of Ordinary Differential Equations and multi-variable calculus.

Course Contents

Unit-1

- Formation of PDEs: First order PDE in two and more independent variables, Derivation of PDE by elimination method of arbitrary constants and arbitrary functions. Lagrange's first order linear PDEs.

Unit-2

- Charpit's method for non-linear PDE of first order, Jacobi's method and Cauchy problem for first order PDEs. PDEs of second order with variable coefficients: Classification of second order PDEs, canonical form, Parabolic, Elliptic and Hyperbolic PDEs.

Unit-3

- Method of separation of variables for Laplace, Heat and Wave equations, General solution of higher order PDEs, Fundamental solution of Laplace Equation.

Unit-4

- Linear integral equation and classification of conditions, Volterra integral equation, Relationship between linear differential equation and Volterra integral equation, Resolvent kernel of Volterra integral equation, solution of integral equation by Resolvent kernel, The method of successive approximation, Convolution type equations.

Unit-5

- Fredholm integral equation, Fredholm equation of the second kind, Fundamentals-iterated kernels, constructing the resolvent kernel with the aid of iterated kernels.

Recommended Texts.

1. K. S. Rao, Introduction to Partial Differential Equations, 3rd Edition, PHI Learning Pvt. Ltd. 2010.
2. A. Jafferey, Applied Partial Differential Equation: An Introduction, Academic Press, 2003.
3. R .P. Kanwal, Linear Integral Equations, Springer Science and Business Media, 2013.

Learning Outcome. The students will be introduced to topics of Partial Differential Equations with an emphasis on type and solutions of different PDEs and types and solutions of different Integral Equations. In particular, students should be able to do the following after completing this course:

1. solve the linear and non-linear PDEs of first and higher dimensions using Lagrange, Charpit and Jacobi methods;
2. able to find the canonical forms of PDEs;
3. learn about parabolic, hyperbolic and elliptic form of PDEs;
4. learn the method of separation and general solutions of Laplace, Heat and Wave equations;
5. learn about linear and Volterra integral equations;
6. able to find the solution of Integral Equations by Resolvent kernel;
7. learn about the Fredholm Integral Equations, Fredholm equation of second kind, iterated kernels;
8. construct the resolvent kernel with the help of iterated kernels.

Course code: MMAT1O013T **Course title:** Introduction to LaTeX

Objective. The main goal is to impart knowledge and understanding about the LaTeX system, explain the procedure of LaTeX typesetting, and familiarize participants with various LaTeX document formats, allowing them to confidently prepare research articles, theses, books, and presentations.

Course Contents

Unit-1

- Installation of the software LATEX, Document class, Adding different packages, Creating and typesetting a LATEX document-sectioning, display material, running LATEX, changing the type style, Elementary and advanced mathematical typesetting-producing mathematical symbols and mathematical formulae, arrays, matrix, multiline formulae, spacing in math mode, Typesetting theorem in latex.

Unit-2

- Creating table, Creating multi-rows and multi-columns, Adjusting table dimensions, handling data in tables, rotation of tables, Including graphics in LATEX file, Creating flow-charts and block diagrams using tikz environment, Numbering and citation of figures and equations.

Unit-3

- Making presentation slides in beamer class LATEX, various styles in beamer presentation, Cross-reference and citations, Making index and glossary, notes and letters, Bibliography-Creating bibtex file, reference management using JabRef, Typesetting of Journal articles, technical reports, thesis, books and slide presentations.

Recommended Texts.

1. Stefan Kottwitz., 2011, LaTeX Beginner's Guide, Packt publishing, U. K.
2. H. Kopka ., P. W. Daly, 1999, A Guideline to LaTeX , 3rd edition, Addison – Wesley, London.

Learning Outcomes. Students will be able to:

1. To understand features of LaTeX.
2. Typeset complex mathematical formulae using LaTeX.
3. Use tabular and array environments within LaTeX.
4. Create or import graphics into a LaTeX document using a variety of methods.
5. Typeset journal articles, technical reports, thesis, books, and slide presentations.
6. Learn automatic generation of table of contents, bibliographies and indexes

Course code: MMAT1O013T **Course title:** Introduction to LaTeX

Objective. The main goal is to impart knowledge and understanding about the LaTeX system, explain the procedure of LaTeX typesetting, and familiarize participants with various LaTeX document formats, allowing them to confidently prepare research articles, theses, books, and presentations.

Course Contents

Unit-1

- Installation of the software LATEX, Document class, Adding different packages, Creating and typesetting a LATEX document-sectioning, display material, running LATEX, changing the type style, Elementary and advanced mathematical typesetting-producing mathematical symbols and mathematical formulae, arrays, matrix, multiline formulae, spacing in math mode, Typesetting theorem in latex.

Unit-2

- Creating table, Creating multi-rows and multi-columns, Adjusting table dimensions, handling data in tables, rotation of tables, Including graphics in LATEX file, Creating flow-charts and block diagrams using tikz environment, Numbering and citation of figures and equations.

Unit-3

- Making presentation slides in beamer class LATEX, various styles in beamer presentation, Cross-reference and citations, Making index and glossary, notes and letters, Bibliography-Creating bibtex file, reference management using JabRef, Typesetting of Journal articles, technical reports, thesis, books and slide presentations.

Recommended Texts.

1. Stefan Kottwitz., 2011, LaTeX Beginner's Guide, Packt publishing, U. K.
2. H. Kopka ., P. W. Daly, 1999, A Guideline to LaTeX , 3rd edition, Addison – Wesley, London.

Learning Outcomes. Students will be able to:

1. To understand features of LaTeX.
2. Typeset complex mathematical formulae using LaTeX.
3. Use tabular and array environments within LaTeX.
4. Create or import graphics into a LaTeX document using a variety of methods.
5. Typeset journal articles, technical reports, thesis, books, and slide presentations.
6. Learn automatic generation of table of contents, bibliographies and indexes

Course code: MMAT1O013T **Course title:** Calculus of Variation

Objective. The goal of this course is to teach about different types of variation problems as well as necessary and sufficient conditions. This course provides a rigorous modern treatment of calculus of variations, blending classical and modern approaches and applications.

Course Contents

Unit-1

- Functional, variation of functional and its properties, fundamental lemma of calculus of variation, Euler's equations, necessary and sufficient conditions for extremum, The Brachistochrone problem, Functionals dependent on higher order derivatives and several dependent variables.

Unit-2

- Variational problems with fixed boundaries, Transversality conditions, Orthogonality conditions, Sturm-Liouville's theorem on extremals, one sided variations, Hamilton's principle, The principle of least action, Lagrange's equations from Hamilton's principle.

Unit-3

- Variational Methods: Direct Methods, Euler's finite difference method, The Ritz method, Kantorovich Method for Boundary value problems in ODE's and PDE's, Isoperimetric Problem.

Recommended Texts.

1. Elsgolts, L., 1977, Differential Equations and the Calculus of Variations, MIR publishers, Moscow.
2. Gelfand, I. M. and Fomin, S. V., 1963, Calculus of Variations, PRENTICE-HALL, INC. Englewood Cliffs, New Jersey.
3. Elsgolc, L. D., 2012, Calculus of Variations, United States: Dover Publications.

Learning Outcomes. Students by the end of this course, will be able to do the following:

1. Apprehend functionals and have some understanding of their applications.
2. Use the formula for determining a functional's stationary paths to derive the differential equations for stationary paths in simple cases.
3. Find differential equations for stationary paths using the Euler-Lagrange equation or its first integral.
4. In simple cases, solve differential equations for stationary paths subject to boundary conditions.

Course code: MMAT1O013T **Course title:** Calculus of Variation

Objective. The goal of this course is to teach about different types of variation problems as well as necessary and sufficient conditions. This course provides a rigorous modern treatment of calculus of variations, blending classical and modern approaches and applications.

Course Contents

Unit-1

- Functional, variation of functional and its properties, fundamental lemma of calculus of variation, Euler's equations, necessary and sufficient conditions for extremum, The Brachistochrone problem, Functionals dependent on higher order derivatives and several dependent variables.

Unit-2

- Variational problems with fixed boundaries, Transversality conditions, Orthogonality conditions, Sturm-Liouville's theorem on extremals, one sided variations, Hamilton's principle, The principle of least action, Lagrange's equations from Hamilton's principle.

Unit-3

- Variational Methods: Direct Methods, Euler's finite difference method, The Ritz method, Kantorovich Method for Boundary value problems in ODE's and PDE's, Isoperimetric Problem.

Recommended Texts.

1. Elsgolts, L., 1977, Differential Equations and the Calculus of Variations, MIR publishers, Moscow.
2. Gelfand, I. M. and Fomin, S. V., 1963, Calculus of Variations, PRENTICE-HALL, INC. Englewood Cliffs, New Jersey.
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Learning Outcomes. Students by the end of this course, will be able to do the following:

1. Apprehend functionals and have some understanding of their applications.
2. Use the formula for determining a functional's stationary paths to derive the differential equations for stationary paths in simple cases.
3. Find differential equations for stationary paths using the Euler-Lagrange equation or its first integral.
4. In simple cases, solve differential equations for stationary paths subject to boundary conditions.

Course Structure for Ph.D. Mathematics

Course Code	Course Title	Lec Hr	Lab Hr	Tut	SSHr	Credits
PMAT1C001T	Commutative Algebra	4	0	1	8	4
PMAT1C002T	Theory of Bergman spaces	4	0	1	8	4
PMAT1C003T	Composition Operators and Geometric function theory	4	0	1	8	4
PMAT1O004T	Stochastic Processes and Queueing Models	4	0	1	8	4

- The candidate has to earn a minimum of 8 credits by selecting one course from the above table and the following two compulsory courses of 2 credits each on SWAYAM:
  (1). Research Methodology
  (2). Research Ethics and Plagiarism,

Course code: PMAT1C001T **Course title:** Commutative Algebra

Objectives. The aim of this course is to introduce the students to the notion of prime and maximal ideals, nilradical and Jacobson radical, extension and contraction of ideals, Colon ideals, Modules and their tensor product, localization of rings and modules, Primary decomposition, Integral dependence, chain conditions, Discrete valuation rings, Hilbert function and Hilbert polynomial.

Course contents

Unit-1

- Rings, ring homomomorphisms, Ideals, Operation on ideals, Quotient rings, Zero-divisors, nilpotents and units, Prime and maximal ideals, Local ring, Nilradical and Jacobson radical, Extension and contraction of ideals, Exercises based on above topics.

Unit-2

- Modules, module homomorphisms, Submodules, Quotient modules, Operation on submodules, direct sum and product of modules, Finitely generated modules: Nakayama Lemma, Tensor product of modules and its exactness properties, Extension and restriction of scalars, Exercises based on the above topics

Unit-3

- Localization of rings and modules, properties of localization, Primary decomposition: Primary ideals, uniqueness of primary decomposition, Exercises based on above topics

Unit-4

- Integral dependence: Transitivity of integral dependence, Going-Up and Going down theorems.
- Chain conditions: Noetherian and Artinian modules, Noetherian rings: Hilbert Basis Theorem, Irreducible ideals and primary decomposition in Noetherian rings, Artin rings, Exercises based on above topics

Unit-5

- Discrete valuation rings and its properties, Dedekind domains and its properties, Hilbert function: Graded rings and modules, Hilbert function and polynomial, Exercises based on above topics

Recommended Texts.

1. M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley Publishing Company, 1994.
2. D. Eisenbud. Commutative Algebra: With a View Toward Algebraic Geometry, Springer-Verlag, New York 1999.
3. E. Kunz, Introduction to Commutative Algebra and Algebraic Geometry, Birkhäuser, 1985
4. M. Reid, Undergraduate Commutative Algebra: London Mathematical Society Student Texts, Cambridge University Press, Cambridge, 1996.

Learning outcome. After completion of this course, the learner will be able to compute prime and maximal ideals in certain commutative rings, classify all finitely generated modules, compute tensor product of certain modules, compute the localization at prime ideals, compute the primary decomposition of ideals, construct rings with chain conditions, write Hilbert function and Hilbert function of certain graded algebras.

Course code: PMAT1C002T **Course title:** Theory of Bergman spaces

Objectives. The aim of this course is to introduce the students to the notion of Bergman spaces, approximation of function in Bergman spaces, orthonormal basis and Bergman kernel, duality of Bergman spaces, Berezin transform, Carleson measures inner functions in Bergman spaces.

Course contents

Unit-1

- Bergman Spaces, Point evaluation as bounded linear functional on Bergman spaces, Bergman spaces as closed subspaces of $L^p(\mathbb{D}, dA)$. Dilation function, Approximation of function in Bergman spaces by dilation functions and by polynomials, Orthonormal basis and reproducing Kernel functions.

Unit-2

- Bergman metric, Duality of Bergman spaces, Equivalent norms of Bergman spaces, Atomic decomposition.

Unit-3

- Basic properties of Bloch spaces, Little Bloch spaces, Analytic Besov spaces and Growth spaces.

Unit-4

- The Berezin transform, Some properties of Berezin transform, Toplitz operators, Toeplitz operators and Berezin transform, Carleson measures and vanishing Carleson measures.

Unit-5

- Definition of inner functions in Bergman spaces, Examples of inner functions, zero set for Bergman spaces, notion of density, Necessary and sufficient conditions for Zero sets of Growth and Bergman spaces.

Recommended Texts.

1. H.Hedenamal, B.Korenblum and K.Zhu, Theory of Bergman spaces, Springer, 2000.
2. K.Zhu, Operator theory in function spaces, CRC press, 1990.

Learning outcome. After completion of this course, the candidate will learn some basic properties of Bergman and peruse independent research in this area.

Course code: PMAT1C003T **Course title:** Composition Operators and Geometric Function Theory

Objectives. The aim of this course is to introduce the students to the notion of Bergman type spaces.

Course contents

Unit-1

- A brief introduction to linear fractional transformations (LFT), fixed points, classifications of LFT.

Unit-2

- The Hardy space H^2; growth estimate, Definition of H^2 via, integral means, Littlewood subordination Principle, definition of composition operators, composition operators induced by automorphism, Littlewood's Theorem.

Unit-3

- Definition of compact operator, finite rank approximation theorem, first compactness theorem, Hilbert-Schmidt Theorem for composition operators, the polygonal compactness theorem, weak compactness theorem for composition operators, non-compact composition operators, comparison principle for compactness.

Unit-4

- Definition of compact operator, finite rank approximation theorem, first compactness theorem, Hilbert-Schmidt Theorem for composition operators, the polygonal compactness theorem, weak compactness theorem for composition operators, non-compact composition operators, comparison principle for compactness.
- Area integral estimate for the H^2 Norm, univalent compactness Theorem Adjoint of composition operators on reproducing Kernel functions, contact of a region in unit disk with the unit circle with examples.

Unit-5

- Definition of Angular Derivative, the Julia Caratheodory Hieodory Theorem, Angular Derivative, Criterion for compactness, the Pseudo-hyperbolic distance. The invariant Schwartz Lemma.

Recommended Texts.

1. Carl. C. Cowen and B. D. MacCluer, composition operators on spaces of analytic functions, CRC Press,Boca Raten, 1995.
2. J. H. Shapiro, Composition Operators and Classical Function Theory, Springer Verlag, 1993.
3. W. Rudin, Real and Complex Analysis, Third Ed. McGraw Hill, New York 1987.

Learning outcome. After completion of this course, the will learn basic properties of Composition operators, geometric function theory and peruse independent research in this area.

Course code: PMAT1C004T **Course title:** Stochastic Processes and Queueing Models

Objectives.

1. To Provide a thorough understanding of the mathematical foundations of Queueing models which are very often seen our daily life such as in machine repair systems, manufacturing/production systems, telecommunication and computer communication systems, etc.
2. To teach the applications of Stochastic and Markov processes with queueing models, to analyze the performance measures for various Markovian and Non-Markovian Queueing models.

Course contents

Unit-1

- Definitions and Examples of Stochastic Processes, Types of Stochastic Processes with examples, Expectation, Covariance, Co-relation, Stationary Processes, Sum of Stochastic Processes and its properties.

Unit-2

- Definitions and Examples of Markov Processes, Markov Chain, Classification of States, Regular Markov Chain, Ergodicity, n-step transition probabilities, Transition Probability Matrix (TPM) and its properties.

Unit-3

- Some Distributions namely Binomial, Geometric, Exponential, Poisson, Erlang, Poisson Processes, Generating functions, Probability Generating Functions (PGF), Expectation and Variance in terms of PGF, Pure Birth Processes, Pure Death Processes, Birth and Death Processes.

Unit-4

- Queueing Systems and its Characteristics, Finite and Infinite capacity Queueing Models: M/M/1, M/M/C, M/M/1/K, M/M/C/K and the evaluation of their Performance Measures.

Unit-5

- Queues with Bulk arrivals and services and its performance evaluation, Some Non-Markovian Queueing Models as M/G/1 etc., Series Queues and its Performance measures.

Recommended Texts.

1. Palaniammal, S., Probability and Queueing Theory, PHI Learning Private Limited, New Delhi, 2012.
2. Medhi, J., Stochastic Processes, New Age International Publishers New Delhi, Third Edition, 2009.
3. Shortle, J. F., Thompson, J. M., Gross, D. and Harris, C. M., Fundamentals of Queueing Theory, Fifth Edition, Willy Series in Probability and Statistics, May, 2018.
4. Kulkarni, Vidyadhar G., Modeling and Analysis of Stochastic Systems, CRC Press, 2016.

Learning outcome.

1. After successfully completing this course, students will acquire the skill of mapping frequently occurring scenarios in machine repair systems, manufacturing/production systems, telecommunication and computer communication systems, etc into standard stochastic models.
2. They will be able to construct mathematical models from the physical description of the problems. Also, they will be able to identify appropriate solution methods in each case and physically interpretation by using mathematical results.

Course code: PMAT1C004T **Course title:** Stochastic Processes and Queueing Models

Objectives.

1. To Provide a thorough understanding of the mathematical foundations of Queueing models which are very often seen our daily life such as in machine repair systems, manufacturing/production systems, telecommunication and computer communication systems, etc.

2. To teach the applications of Stochastic and Markov processes with queueing models, to analyze the performance measures for various Markovian and Non-Markovian Queueing models.

Course contents

Unit-1

- Definitions and Examples of Stochastic Processes, Types of Stochastic Processes with examples, Expectation, Covariance, Co-relation, Stationary Processes, Sum of Stochastic Processes and its properties.

Unit-2

- Definitions and Examples of Markov Processes, Markov Chain, Classification of States, Regular Markov Chain, Ergodicity, n-step transition probabilities, Transition Probability Matrix (TPM) and its properties.

Unit-3

- Some Distributions namely Binomial, Geometric, Exponential, Poisson, Erlang, Poisson Processes, Generating functions, Probability Generating Functions (PGF), Expectation and Variance in terms of PGF, Pure Birth Processes, Pure Death Processes, Birth and Death Processes.

Unit-4

- Queueing Systems and its Characteristics, Finite and Infinite capacity Queueing Models: M/M/1, M/M/C, M/M/1/K, M/M/C/K and the evaluation of their Performance Measures.

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- Queues with Bulk arrivals and services and its performance evaluation, Some Non-Markovian Queueing Models as M/G/1 etc., Series Queues and its Performance measures.

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