DC circuit analysis

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RC circuit

- Consider a RC series circuit with resistor R and capacitor C connected with a constant voltage source V.

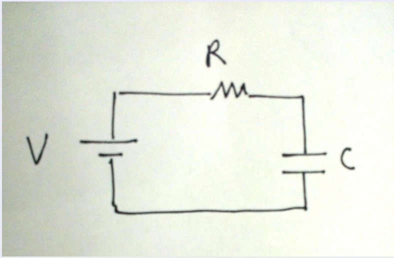

Figure 1: A typical RC series circuit.

Analysis

- Using Kirchoff's voltage law, we write

$$\begin{aligned} V &= V_R + V_C, \\ V &= RI + q/C, \\ V &= R\frac{dq}{dt} + \frac{q}{C}. \end{aligned} \tag{1}$$

- Rearranging, we can write

$$\frac{dq}{dt} + \frac{q}{\tau} = \frac{V}{R}, \tag{2}$$

  where $\tau = RC$ is the characteristic time constant.

- This is a first order linear ordinary differential equation.

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- Solving the Eq. (2), we find charge as a function of time

$$q(t) = CV[1 - \exp(-t/\tau)]. \quad (3)$$

- The corresponding current decreases exponentially as

$$I(t) = \frac{dq}{dt} = \frac{V}{R}\exp(-t/\tau). \quad (4)$$

- The voltage across capacitor increases as

$$V_C = \frac{q}{C} = V[1 - \exp(-t/\tau)]. \quad (5)$$

RL circuit

- Consider a RL series circuit with resistor R and inductor L connected with a constant voltage source V. A similar analysis can be done here.
- The current as a function of time increases as

$$I(t) = \frac{V}{R}[1 - \exp(-t/\tau)], \tag{6}$$

  where $\tau = L/R$.
- The voltage across inductor decreases as

$$V_L = V \exp(-t/\tau). \tag{7}$$