Syllabus for M.Sc. Mathematics

Course Code	Course Title	L-P-T	Credits
MMAT1C005T	Complex Analysis	3-0-1	4

Objective: The objectives of this Course is to introduce the fundamental ideas of the functions of Complex variables and developing a clear understanding of the fundamental concepts of complex Analysis such an Analytic functions, Complex integral s and arrange of skills which will allow students to work effectively with the concepts

CO 01	**Understand Fundamental Concepts** • Demonstrate a comprehensive understanding of complex numbers and their geometric representation. • Identify and classify different types of complex functions and their properties.
CO 02	**Analytical Techniques:** • Apply the Cauchy-Riemann equations to determine analytic functions. • Solve complex integrals using Cauchy's integral theorem and Cauchy's integral formula.
CO 03	**Residue Theory:** • Utilize the residue theorem to evaluate real integrals and compute residues at isolated singularity. • Analyze complex functions to identify poles, essential singularities, and removable singularities.
CO 04	**Series Expansions:** • Derive and apply Taylor and Laurent series to represent complex functions. • Use power series to understand the convergence of complex functions within their radius of convergence
CO 05	**Mapping and Transformations:** • Explore conformal mappings and their applications in solving boundary value problems. • Analyze the effects of various transformations on the complex plane.

Course Content

Unit-1

Review of complex numbers, Stereographic projection, Chordal distance, Multi-valued functions, Branches of multi-valued functions, with special reference to *arg z*, exponential functions, Logarithm function, power functions and phase factors. Analytic functions: Limit and continuity of complex functions, complex derivative, Singularities, Cauchy- Tiemann equations, Cauchy-Reinmann equations in polar form, Harmonic functions, and Harmonic conjugate.

Unit-2

Line integrals, Piecewise smooth path, Jordan curve, Green's theorem, Independence of path, Anti-derivative, fundamental theorem of calculus, Mean value property, Strict maximum principal (real and complex version), ML-estimate.

Kamlesh Kumar

Shalaksh

Unit-3

Complex integration and analyticity: Cauchy's theorem, Cauchy Integral formula, Cauchy integral formulae for higher order derivatives.

Liouville's theorem, Cauchy's inequality, Morera's theorem, Goursat's theorem, complex form of Cauchy-Riemann equations.

Unit-4

Power series, radius of convergence, power series expansion of an analytic function: Taylor's expansion, isolated singularities, Laurent Series. The residue calculus, Cauchy residue theorem, fractional residues, Jordan's lemma, Evaluation of integrals using residue theorem.

Unit-5

Conformal mappings, Mobius transformations, composition of two Mobius transformations Translations, Dilations, Inversion, The Schwarz lemma, Conformal Self-maps of the unit disk, Mappings of the unit disk and upper half plane, The Riemann Mapping theorem (Statement only).

Reference Books:

1. TW Gamelin, Complex Analysis, Springer–Verlag, New York Berlin Heidelberg 2001.
2. Walter Rudin; Real & Complex Analysis, Tata Mc-Graw Hill, 2006
3. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2005
4. J.W. Brown & R.V. Churchill, Complex variables and applications, Mc-Graw Hill International VIII-Edition, 2009
5. J.B. Conway, Function of One complex variable, Springer International Student Edition, 1980
6. L.V. Ahlfors, Complex Analysis, International Edition, McGraw Hill International Editions, 1979.