Course No.	Course Title	L-P-T	Credits
UMAT0O0022T	**Operator Theory**	3-0-1	4

OBJECTIVES: This course introduces fundamental concepts of operator theory, including spectrum, normed and Banach algebras, and compact operators. It highlights applications in differential equations, quantum mechanics, and various mathematical fields. A basic understanding of functional analysis is assumed, but no prior knowledge of operator theory is required.

CO 01	Understand spectral theory in finite and infinite normed spaces, including eigenvalues, resolvent operators, and spectral mapping theorem applications.
CO 02	Analyze Banach algebras, invertible elements, spectral properties, and key theorems like Banach-Alaoglu and Gelfand-Mazur.
CO 03	Study compact linear operators, their eigenvalues, eigenspaces, compactness criteria, and uniform limits in normed linear spaces.
CO 04	Explore unbounded linear operators, adjoint operators, closed operators, and spectral properties of self-adjoint linear operators.
CO 05	Examine multiplication and differentiation operators, quantum mechanics concepts like states, observables, and Heisenberg's uncertainty principle.

Course contents

Unit-1

Spectral theory in Finite normed spaces: Definition of eigenvalues, eigen vectors, eigen spaces, spectrum, eigenvalues of an operator, Existence Theorem for eigenvalues, spectral theory for infinite dimensional normed linear spaces, resolvent of an operator, spectrum of a bounded linear operator on complex Banach space, Representation Theorem, Resolvent equation, commutative properties of resolvent , Spectral Mapping theorem for polynomials (statement only), definition of local holomorphy, holomorphy of resolvent operator, spectral radius of a bounded linear operator.

Unit-2

Definition of normed algebra, Banach Algebra and examples, invertible elements, Banach-Alaoglu theorem (statement only), multiplicative linear functional, definition of spectrum, resolvent set, spectral radius, division algebra, Gelfand Mazur Theorem, Spectral Mapping Theorem.

Unit-3

Definition of compact linear operator on normed linear spaces, examples, compactness criterion, uniform limit of a sequence of compact operators, finite rank operator, eigenvalues and eigenspaces for compact operators.

Unit-4

Unbounded Linear Operators: Hellinger-Toeplitz Theorem, Densely defined operators, Hilbert-Adjoint operators, Inverse of the Hilbert-adjoint operator, Symmetric linear operator, closed linear operator, definition of closable operator, closure, spectrum of self-adjoint linear operator.

Unit-5

Multiplication operator and Differentiation operator, self-adjoint multiplication operator, spectrum of multiplication operator, definitions of States, Observables, Position operator and Moment operator, Heisenberg Uncertainty Principle.

Kamlesh Kumar

Shalabh