

# **THRESHOLD F-POLICY AND N-POLICY FOR REDUNDANT MACHINING SYSTEM**

**By**

**Dr. Kamlesh Kumar**



**DEPARTMENT OF MATHEMATICS  
CENTRAL UNIVERSITY OF JAMMU**

# **THRESHOLD F-POLICY AND N-POLICY FOR REDUNDANT MACHINING SYSTEM**

- We develop a Markovian redundant machining system for F-policy and N-policy by using birth & death process. To formulate the mathematical model, we construct the steady state governing equations in terms of probabilities by using the appropriate rates of in-flow and out-flow.
  
- We develop  $(m, M)$  model for multi-component system under the assumption that the system fails when there are  $L=M+S-m+1$  ( $m=1, 2, \dots, M$ ) or more failed machines/units in the system. We use recursive method to solve the steady state governing equations and performance indices for both policies models.

# Model Description

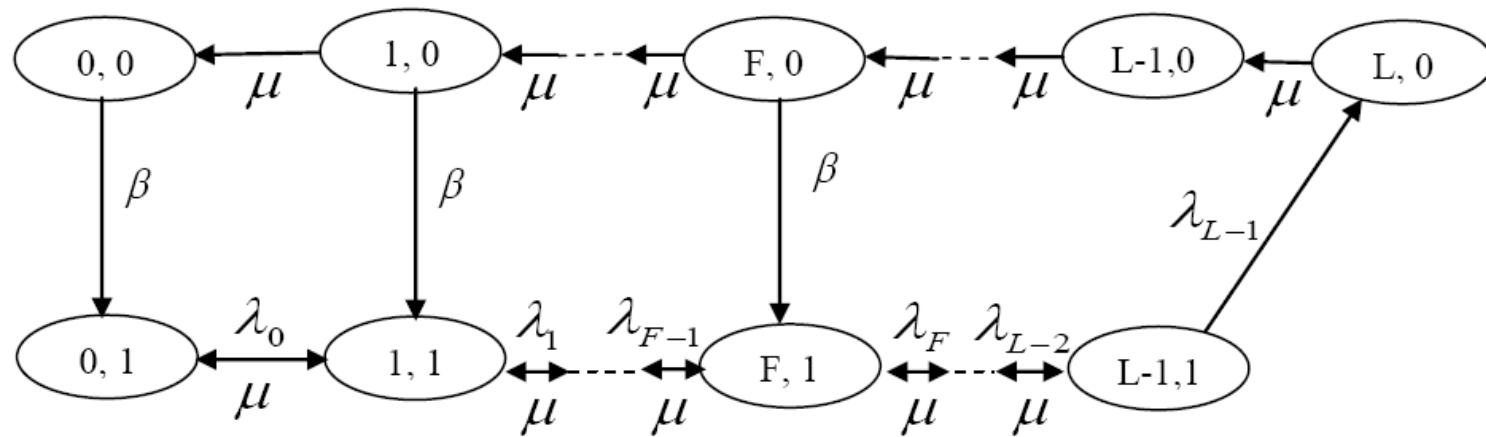


Fig. 6.1: State transition diagram for  $F$ -policy model

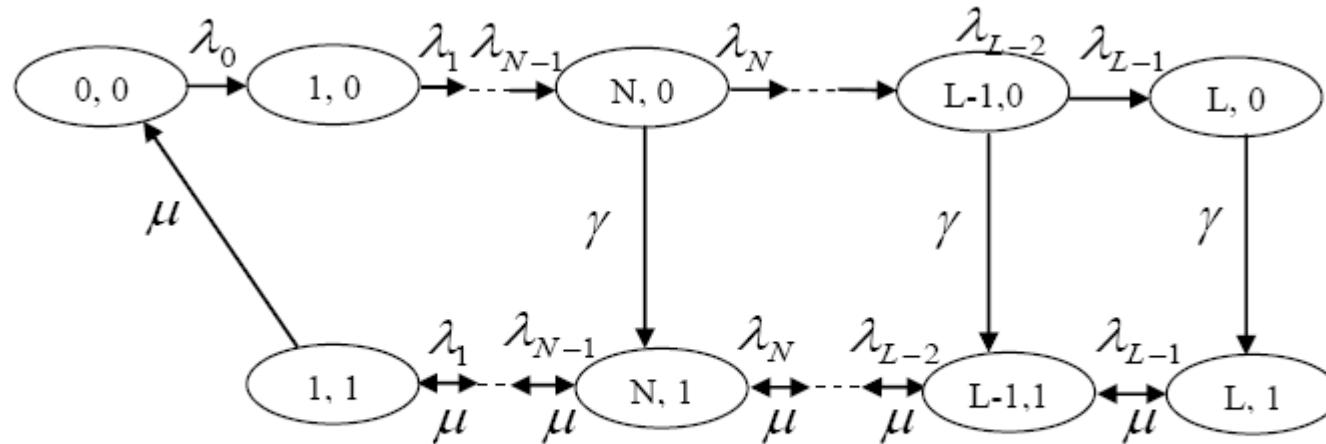


Fig. 6.2: State transition diagram for  $N$ -policy model

The state dependent probabilities for both models are stated as below:

$P_{n,j}$  : Probability that there are  $n$  failed machines in the system and the failed machines are either allowed ( $j=1$ ) or not allowed ( $j=0$ ) for repair in case of  $F$ -policy model.

$Q_{n,j}$  : Probability that there are  $n$  failed machines in the system and the server is either busy ( $j=1$ ) or idle ( $j=0$ ) in case of  $N$ -policy model.

# Governing Equations

The state dependent failure rate is defined by

$$\lambda_n = \begin{cases} M\lambda + (S-n)\alpha; & 0 \leq n < S \\ (M+S-n)\lambda_d; & 0 \leq n < S \\ 0; & \text{otherwise} \end{cases}$$

The steady state difference equations for F-Policy model can be defined as:

❖ For J=0, when failed machines are not allowed in the system.

$$\mu P_{1,0} = \beta P_{0,0} \quad \dots(6.1),$$

$$\mu P_{n+1,0} = (\mu + \beta) P_{n,0}; \quad 1 \leq n \leq F \quad \dots(6.2)$$

$$P_{n+1,0} = P_{n,0}; \quad F+1 \leq n \leq L-1 \quad \dots(6.3),$$

$$\lambda_{L-1} P_{L-1,0} = \mu P_{L,0} \quad \dots(6.4)$$

❖ For J=1, when failed units are allowed for repair in the system.

$$P_{1,1} + \beta P_{0,0} = \lambda_0 P_{0,1} \quad \dots(6.5)$$

$$\lambda_{n-1} P_{n-1,1} + \mu P_{n+1,1} + \beta P_{n,0} = (\lambda_n + \mu) P_{n,1}; \quad 1 \leq n \leq F \quad \dots(6.6)$$

$$\lambda_{n-1} P_{n-1,1} + \mu P_{n+1,1} = (\lambda_n + \mu) P_{n,1}; \quad F+1 \leq n \leq L-2 \quad \dots(6.7)$$

$$\lambda_{L-2} P_{L-2,1} = (\lambda_{L-1} + \mu) P_{L-1,1}; \quad F \neq L-1 \quad \dots(6.8)$$

The normalization condition is given by  $\sum_{j=0}^1 \sum_{n=0}^{L-1} P_{n,j} + P_{L,0} = 1$ , ... (6.9)

**Eqns (6.1)-(6.3) have been solved recursively and steady state probability can be obtained as:**

$$P_{n,0=\delta(1+\delta)^{n-1} P_{0,0}}; 1 \leq n \leq F \quad ....(6.10),$$

$$P_{n,0=\delta(1+\delta)^F} P_{0,0}; F+1 \leq n \leq L \quad \dots\dots (6.11), \quad \text{where} \quad \delta = \frac{\beta}{\mu}$$

$$P_{L-1,1} = \frac{\mu\delta(1+\delta)^F}{\lambda_{L-1}} P_{0,0} \quad \dots (6.12)$$

$$P_{L-2,1} = \frac{(\mu + \lambda_{L-1})\mu\delta(1+\delta)^F}{\lambda_{L-1}\lambda_{L-2}} P_{0,0} \quad \dots(6.13)$$

$$P_{n,1} = \frac{\delta_F}{\prod_{i=n}^{L-1} \lambda_i} \left[ \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=m+F}^{L-1} \lambda_i \right\} \right]; F \leq n \leq L-1 \quad \dots(6.14)$$

❖ Product Formula may be defined as:

$$\prod_{i=p}^{p+1} \lambda_i; \text{ for any number } p \quad (6.15)$$

❖ In equation (6.8), we put  $n=F, F+1, F+2, \dots, 1$ , and get :

$$P_{F-1,1} = \frac{\delta_F}{\prod_{i=F-1}^{L-1} \lambda_i} \left[ \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=m+F}^{L-1} \lambda_i \right\} \right] - \frac{\delta \delta_F}{\lambda_{F-1}(1+\delta)} \quad (6.15a)$$

$$P_{F-2,1} = \frac{\delta_F}{\prod_{i=F-2}^{L-1} \lambda_i} \left[ \sum_{m=0}^{L-(F-1)} \left\{ \mu^m \prod_{i=m+F-1}^{L-1} \lambda_i \right\} \right] - \frac{(\mu + \lambda_{F-1}) \delta \delta_F}{\lambda_{F-1} \lambda_{F-2} (1+\delta)} - \frac{\delta \delta_F}{\lambda_{F-2} (1+\delta)^2} \quad (6.15b)$$

$$P_{F-3,1} = \frac{\delta_F}{\prod_{i=F-3}^{L-1} \lambda_i} \left[ \sum_{m=0}^{L-(F-2)} \left\{ \mu^m \prod_{i=m+F-2}^{L-1} \lambda_i \right\} \right] - \frac{(\lambda_{F-2} \lambda_{F-1} + \mu \lambda_{F-1} + \mu^2) \delta \delta_F}{\lambda_{F-3} \lambda_{F-1} \lambda_{F-2} (1+\delta)} - \frac{(\mu + \lambda_{F-2}) \delta \delta_F}{\lambda_{F-3} \lambda_{F-2} (1+\delta)^2} - \frac{\delta \delta_F}{\lambda_{F-3} (1+\delta)^3} \quad (6.15c)$$

$$P_{0,1} = \frac{\delta_F}{\prod_{i=1}^{L-1} \lambda_i} \left[ \sum_{m=0}^{L-2} \left\{ \mu^m \prod_{i=m+2}^{L-1} \lambda_i \right\} \right] - \frac{\delta \delta_F}{(1+\delta) \prod_{i=1}^{F-1} \lambda_i} \left[ \sum_{m=0}^{F-2} \left\{ \mu^m \prod_{i=m+2}^{L-1} \lambda_i \right\} \right] - \frac{\delta \delta_F}{(1+\delta)^{F-2} \prod_{i=1}^2 \lambda_i} \left[ \sum_{m=0}^1 \left\{ \mu^m \prod_{i=m+2}^{L-1} \lambda_i \right\} \right] - \frac{\delta \delta_F}{\lambda_1 (1+\delta)^{F-1}} \quad (6.15d)$$

$$P_{n,1} = \frac{\delta_F}{\prod_{i=n}^{L-1} \lambda_i} \left[ \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] - \delta_F \delta \sum_{k=1}^{F-n} \left\{ \frac{\sum_{m=0}^{F-n-k} \left( \mu^m \prod_{i=n+m+1}^{F-k} \lambda_i \right)}{(1+\delta)^k \prod_{i=n}^{F-k} \lambda_i} \right\} . \quad (6.16)$$

$$P_{0,0^{-1}} = (1+\delta)^F \{1+\delta(L-F)\} + \mu \delta (1+\delta)^F \sum_{n=0}^{L-1} \left[ \frac{1}{\prod_{i=n}^{L-1} \lambda_i} \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] - \mu \delta^2 (1+\delta)^F \sum_{n=0}^{F-1} \left[ \sum_{k=1}^{F-n} \left\{ \frac{\sum_{m=0}^{F-n-k} \left( \mu^m \prod_{i=n+m+1}^{F-k} \lambda_i \right)}{(1+\delta)^k \prod_{i=n}^{F-k} \lambda_i} \right\} \right] \dots \quad (6.17)$$

# The steady state difference equations for N-Policy can be defined as:

❖ For j=0, when the server is idle

$$\mu Q_{1,1} = \lambda_0 Q_{0,0} \quad \dots(6.18)$$

$$\lambda_n Q_{n,0} = \lambda_{n-1} Q_{n-1,0}; 1 \leq n \leq N-1 \quad \dots(6.19)$$

$$(\lambda_n + \gamma) Q_{n,0} = \lambda_{n-1} Q_{n-1,0}; N \leq n \leq L-1 \dots(6.20)$$

$$\gamma Q_{L,0} = \lambda_{L-1} Q_{L-1} \quad \dots(6.21)$$

❖ For j=1, when server is busy.

$$(\lambda_1 + \mu) Q_{1,1} = \mu Q_{2,1}; N \neq 1 \quad \dots(6.22)$$

$$(\lambda_n + \mu) Q_{n,1} = \lambda_{n-1} Q_{n-1,1} + \mu Q_{n+1,1}; 2 \leq n \leq N-1 \quad \dots(6.23)$$

$$(\lambda_n + \mu) Q_{n,1} = \lambda_{n-1} Q_{n-1,1} + \mu Q_{n+1,1} + \gamma Q_{n,0}; N \leq n \leq L-1 \dots(6.24)$$

$$\gamma Q_{L,1} = \lambda_{L-1} Q_{L-1,1} + \gamma Q_{L,0} \quad \dots(6.25)$$

The normalize conditions is given by

$$Q_{0,0} + \sum_{j=0}^1 \sum_{n=0}^L Q_{n,j} = 1, \quad (6.26)$$

Equations (6.18)-(6.26) have solved recursively and steady state probability as:

$$Q_{n,0} = \frac{\lambda_0}{\lambda_n} Q_{0,0}; \quad 1 \leq n \leq N-1 \quad (6.27)$$

Now in Equations (6.20), we put  $n= N, N+1, N+2, \dots, L-1$ , and we get :

$$Q_{n,0} = \frac{\lambda_0 Q_{0,0}}{\lambda_n} \prod_{i=N}^n \theta_i; \quad N \leq n \leq L-1 \quad \dots \dots (6.28), \text{ where } \theta_i = \frac{\lambda_i}{\lambda_i + \gamma}$$

$$Q_{L,0} = \frac{\lambda_{L-1} Q_{L-1,0}}{\gamma} = \frac{\theta_0 Q_{0,0}}{1 - \theta_0} \prod_{i=N}^n \theta_i \quad \dots \dots (6.29)$$

$$Q_{1,1} = \frac{\lambda_0 Q_{0,0}}{\mu} \quad \dots \dots (6.30)$$

$$Q_{2,1} = \frac{\lambda_0 (\lambda_1 + \mu) Q_{0,0}}{\mu^2} \quad \dots \dots (6.31)$$

$$Q_{n,1} = \frac{\lambda_0 Q_{0,0}}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m-1}^{n-1} \lambda_i \right\}; \quad 1 \leq n \leq N \quad \dots \dots (6.32)$$

$$Q_{N+1,1} = \frac{\lambda_0 Q_{0,0}}{\mu^{N+1}} \sum_{m=0}^N \left\{ \mu^m \prod_{i=m+1}^N \lambda_i \right\} - \frac{\lambda_0 Q_{0,0}}{\mu} (1 - \theta_N) \dots \quad (6.33a)$$

$$Q_{N+2,1} = \frac{\lambda_0 Q_{0,0}}{\mu^{N+2}} \sum_{m=0}^{N+1} \left\{ \mu^m \prod_{i=m+1}^{N+1} \lambda_i \right\} - \frac{\lambda_0 Q_{0,0}}{\mu^2} \{ \lambda_{N+1} (1 - \theta_N) + \mu (1 - \theta_N \theta_{N+1}) \} \dots \quad (6.33b)$$

$$Q_{N+3,1} = \frac{\lambda_0 Q_{0,0}}{\mu^{N+3}} \sum_{m=0}^{N+2} \left\{ \mu^m \prod_{i=m+1}^{N+2} \lambda_i \right\} - \frac{\lambda_0 Q_{0,0}}{\mu^3} \{ \lambda_{N+2} \lambda_{N+1} (1 - \theta_N) + \mu \lambda_{N+2} (1 - \theta_N \theta_{N+1}) + \mu^2 (1 - \theta_N \theta_{N+1} \theta_{N+2}) \} \dots \quad (6.33c)$$

$$Q_{N+4,1} = \frac{\lambda_0 Q_{0,0}}{\mu^{N+4}} \sum_{m=0}^{N+3} \left\{ \mu^m \prod_{i=m+1}^{N+3} \lambda_i \right\} - \frac{\lambda_0 Q_{0,0}}{\mu^3} \{ \lambda_{N+3} \lambda_{N+2} \lambda_{N+1} (1 - \theta_N) + \mu \lambda_{N+3} \lambda_{N+2} (1 - \theta_N \theta_{N+1}) \dots \\ + \mu^2 \lambda_{N+3} (1 - \theta_N \theta_{N+1} \theta_{N+2}) + \mu^3 (1 - \theta_N \theta_{N+1} \theta_{N+2} \theta_{N+3}) \} \dots \quad (6.33d)$$

.....

$$Q_{L,1} = \frac{\lambda_0 Q_{0,0}}{\mu^L} \sum_{m=0}^{L-1} \left\{ \mu^m \prod_{i=m+1}^{L-1} \lambda_i \right\} - \frac{\lambda_0 Q_{0,0}}{\mu^{L-N}} \{ \lambda_{L-1} \dots \lambda_{N+2} \lambda_{N+1} + \mu \lambda_{L-1} \dots \lambda_{N+2} (1 - \theta_N \theta_{N+1}) + \dots \\ + \mu^{L-N-1} (1 - \theta_N \theta_{N+1} \theta_{N+2} \dots \theta_{L-1}) \} \dots \quad (6.33e)$$

$$Q_{n,1} = \frac{\lambda_0 Q_{0,0}}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} - \mu^N \left[ \sum_{m=N}^{n-1} \left\{ \mu^{m-N} \left( 1 - \prod_{i=N}^m \theta_i \right) \right\} \prod_{i=m+1}^{n-1} \lambda_i \right]; N+1 \leq n \leq L \dots \quad (6.34)$$

$$Q_{0,0}^{-1} = \sum_{n=0}^{N-1} \left( 1 + \frac{\lambda_0}{\lambda_n} \right) + \sum_{n=N}^{L-1} \left( \frac{\lambda_0}{\lambda_n} \prod_{i=N}^m \theta_i \right) + \frac{\theta_0}{1 - \theta_0} \prod_{i=N}^{L-1} \theta_i + \sum_{n=1}^L \left[ \frac{\lambda_0}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] \\ - \sum_{n=N+1}^L \left[ \frac{\lambda_0}{\mu^{n-N}} \left[ \sum_{m=N}^{n-1} \left\{ \mu^{m-N} \left( 1 - \prod_{i=N}^m \theta_i \right) \right\} \prod_{i=m+1}^{n-1} \lambda_i \right] \right] \dots \quad (6.35)$$

# Performance Measures for F-policy

- Expected number of failed units in the system is

$$E(N_F) = \sum_{n=1}^L n P_{n,0} + \sum_{n=1}^{L-1} n P_{n,1} = \left[ \frac{\{1-(1+\delta)^F(1-F\delta)\}}{\delta} + \frac{(L-F)(L+F+1)}{2} \delta(1+\delta)^F + \mu\delta(1+\delta)^F \left[ \sum_{n=1}^{L-1} \left[ \frac{1}{\prod_{i=n}^{L-1} \lambda_i} \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] \right] \right] \quad ..(6.36)$$

- The probability that the server being idle is

$$P(I_F) = P_{0,0} \left[ \frac{1}{\prod_{i=0}^{L-1} \lambda_i} \sum_{m=0}^{L-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} - \delta \sum_{k=1}^F \frac{\sum_{m=0}^{F-n-k} \left( \mu^m \prod_{i=n+m+1}^{F-k} \lambda_i \right)}{(1+\delta)^k \prod_{i=n}^{F-k} \lambda_i} \mu\delta(1+\delta)^F + 1 \right] \quad ..(6.37)$$

- The probability that the server being busy

$$P(I_F) = P_{0,0} \left[ \mu\delta(1+\delta)^F \left[ \sum_{n=1}^{L-1} \left[ \frac{1}{\prod_{i=n}^{L-1} \lambda_i} \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] - \delta \sum_{n=1}^{F-1} \sum_{k=1}^F \frac{\sum_{m=0}^{F-n-k} \left( \mu^m \prod_{i=n+m+1}^{F-k} \lambda_i \right)}{(1+\delta)^k \prod_{i=n}^{F-k} \lambda_i} \right] + [(1+\delta)^F \{1+\delta(1-F)\} - 1] \right] \quad ..(6.38)$$

- The probability that the server takes startup time before starting the service to the failed units is

$$P(ST_F) = \sum_{n=0}^F P_{n,0} = (1+\delta)^F P_{0,0} \quad ..(6.39)$$

- The probability that the system is blocked (i.e. the failed unit is not allowed to join the queue).

$$P(SB_F) = \sum_{n=0}^L P_{n,0} = (1+\delta)^F [1 + (L-F)\delta] P_{0,0} \quad ..(6.40)$$

- The probability of build up state is obtained as

$$P(SB_F) = \sum_{n=F+1}^L P_{n,0} = (1+\delta)^F (L-F)\delta P_{0,0} \quad ..(6.41)$$

• The expected number of operating units in the system is obtained for two cases as:

❖ Case-1: When  $S < F$  : 
$$E(O_F) = M - \left[ \frac{(1+\delta)^S}{\delta} [1 - \{1 - \delta(F-S)\}(1+\delta)^{F-S}] - \frac{\delta}{2}(1+\delta)^F(L-F-1)(L+F-2S) \right] + \mu\delta(1+\delta)^F \left[ \sum_{n=S+1}^{L-1} \left[ \frac{(n-S)}{\prod_{i=n}^{L-1} \lambda_i} \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] - \delta \sum_{n=S+1}^{F-1} \sum_{k=1}^F \frac{\sum_{m=0}^{F-n-k} \left( \mu^m \prod_{i=n+m+1}^{F-k} \lambda_i \right)}{(1+\delta)^k \prod_{i=n}^{F-k} \lambda_i} \right] \dots\dots(6.42)$$

❖ Case-2: When  $S > F$  :

$$E(O_F) = M - \left[ \frac{\delta}{2} \{(1+\delta)^F(L-S)(L-S+1)\} + \mu\delta(1+\delta)^F \left[ \sum_{n=S+1}^{L-1} \left[ \frac{(n-S)}{\prod_{i=n}^{L-1} \lambda_i} \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] \right] \right] \dots\dots(6.43)$$

• The throughput of the F-policy system is obtained as

$$Th_F = \sum_{n=1}^L \mu P_{n,0} + \sum_{n=1}^{L-1} \mu P_{n,1} \quad (6.44)$$

• The expected number of warm standby units in the system is obtained as follows:

❖ Case-1: When  $S < F$  : 
$$E(O_F) = M - \left[ \frac{(1+\delta)^S}{\delta} [1 - \{1 - \delta(F-S)\}(1+\delta)^{F-S}] - \frac{\delta}{2}(1+\delta)^F(L-F-1)(L+F-2S) \right] + \mu\delta(1+\delta)^F \left[ \sum_{n=S+1}^{L-1} \left[ \frac{(n-S)}{\prod_{i=n}^{L-1} \lambda_i} \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] - \delta \sum_{n=S+1}^{F-1} \sum_{k=1}^F \frac{\sum_{m=0}^{F-n-k} \left( \mu^m \prod_{i=n+m+1}^{F-k} \lambda_i \right)}{(1+\delta)^k \prod_{i=n}^{F-k} \lambda_i} \right] \dots\dots(6.45)$$

❖ Case-2: When  $S > F$  :

$$E(O_F) = M - \left[ \frac{\delta}{2} \{(1+\delta)^F(L-S)(L-S+1)\} + \mu\delta(1+\delta)^F \left[ \sum_{n=S+1}^{L-1} \left[ \frac{(n-S)}{\prod_{i=n}^{L-1} \lambda_i} \sum_{m=0}^{L-n-1} \left\{ \mu^m \prod_{i=n+m+1}^{L-1} \lambda_i \right\} \right] \right] \right] \dots\dots(6.46)$$

# Performance Measures for N-policy

- Expected number of failed units in the system is

$$E(N_N) = \left[ \sum_{n=1}^{N-1} n \frac{\lambda_0}{\lambda_n} + \sum_{n=N}^{L-1} n \frac{\lambda_0}{\lambda_n} \prod_{i=N}^n \theta_i + \frac{L\theta_0}{1-\theta_0} \prod_{i=N}^n \theta_i \right] + \sum_{n=1}^L \left[ \frac{\mu\lambda_0}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] - \sum_{n=N+1}^L \left[ \frac{\mu\lambda_0}{\mu^{n-N}} \sum_{m=N}^{n-1} \left\{ \mu^{m-N} \left( 1 - \prod_{i=m+1}^{n-1} \theta_i \right) \prod_{i=m+1}^{n-1} \lambda_i \right\} \right]. \quad (6.47)$$

- The probability that the server being idle is

$$P(I_N) = \sum_{n=0}^L Q_{n,0} = Q_{0,0} \left[ \sum_{n=0}^{N-1} \left( 1 + \frac{\lambda_0}{\lambda_n} \right) + \sum_{n=N}^{L-1} \left( \frac{\lambda_0}{\lambda_n} \prod_{i=N}^n \theta_i \right) + \frac{\theta_0}{1-\theta_0} \prod_{i=N}^n \theta_i \right]. \quad (6.48)$$

- The probability that the server being busy

$$P(B_N) = \sum_{n=1}^L Q_{n,1} = Q_{0,0} \left[ \sum_{n=1}^L \left[ \frac{\lambda_0}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] - \sum_{n=N+1}^L \left[ \frac{\lambda_0}{\mu^{n-N}} \left[ \sum_{m=M}^{n-1} \left\{ \mu^{m-N} \left( 1 - \prod_{i=N}^m \theta_i \right) \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] \right] \right]. \quad (6.49)$$

- The probability that the server takes startup time before starting the service to the failed units and to build up state is

$$P(ST_N) = \sum_{n=N}^L Q_{n,0} = Q_{0,0} \left[ \sum_{n=N}^{L-1} \left( \frac{\lambda_0}{\lambda_n} \prod_{i=N}^n \theta_i \right) + \frac{\theta_0}{1-\theta_0} \prod_{i=N}^n \theta_i \right]. \quad (6.50), \quad P(BS_N) = \sum_{n=1}^{N-1} Q_{n,0} = \lambda_0 Q_{0,0} \sum_{n=1}^{N-1} \frac{1}{\lambda_n}. \quad (6.51)$$

- The expected number of operating units in the system is obtained for two cases:

❖ Case-1: When  $S < N$  :

$$\begin{aligned} P(O_N) = & \left[ \sum_{n=S+1}^{N-1} (n-S) \frac{\lambda_0}{\lambda_n} + \sum_{n=N}^{L-1} (n-S) \frac{\lambda_0}{\lambda_n} \prod_{i=N}^n \theta_i + \frac{(L-S)\theta_0}{1-\theta_0} \prod_{i=N}^n \theta_i \right] \\ & + \sum_{n=1}^L \left[ \frac{(n-S)\lambda_0}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] - \sum_{n=N+1}^L \left[ \frac{(n-S)\lambda_0}{\mu^{n-N}} \sum_{m=N}^{n-1} \left\{ \mu^{m-N} \left( 1 - \prod_{i=m+1}^{n-1} \theta_i \right) \prod_{i=m+1}^{n-1} \lambda_i \right\} \right]. \end{aligned} \quad (6.52)$$

❖ Case-2: When  $S \geq N$  :

$$P(O_N) = \left[ \sum_{n=S+1}^{N-1} (n-S) \frac{\lambda_0}{\lambda_n} \prod_{i=N}^n \theta_i + \frac{(L-S)\theta_0}{1-\theta_0} \prod_{i=N}^n \theta_i + \sum_{n=S+1}^L \left[ \frac{(n-S)\lambda_0}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] \right] - \sum_{n=S+1}^L \left[ \frac{(n-S)\lambda_0}{\mu^{n-N}} \sum_{m=N}^{n-1} \left\{ \mu^{m-N} \left( 1 - \prod_{i=m+1}^{n-1} \theta_i \right) \prod_{i=m+1}^{n-1} \lambda_i \right\} \right]. \quad (6.53)$$

- The expected number of warm standby units in the system is determined for two cases as follows:

❖ Case-1: When  $S < N$  :

$$P(S_N) = Q_{0,0} \left[ \sum_{n=1}^{S-1} (n-S) \frac{\lambda_0}{\lambda_n} - \sum_{n=1}^{S-1} \left[ \frac{(n-S)\lambda_0}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] \right] \dots \quad (6.54)$$

❖ Case-1: When  $S \geq N$  :

$$P(S_N) = Q_{0,0} \left[ \sum_{n=1}^{N-1} (n-S) \frac{\lambda_0}{\lambda_n} + \sum_{n=N}^{S-1} (n-S) \frac{\lambda_0}{\lambda_n} \prod_{i=N}^n \theta_i + \sum_{n=1}^{S-1} \left[ \frac{(n-S)\lambda_0}{\mu^n} \sum_{m=0}^{n-1} \left\{ \mu^m \prod_{i=m+1}^{n-1} \lambda_i \right\} \right] \right] - \sum_{n=N+1}^{S-1} \left[ \frac{(n-S)\lambda_0}{\mu^{n-N}} \sum_{m=N}^{n-1} \left\{ \mu^{m-N} \left( 1 - \prod_{i=m+1}^{n-1} \theta_i \right) \prod_{i=m+1}^{n-1} \lambda_i \right\} \right]. \quad (6.55)$$

- Throughput of the N-policy system is

$$Th_N = \sum_{n=1}^L \mu Q_{n,1} \quad (6.56)$$

# Numerical Illustration

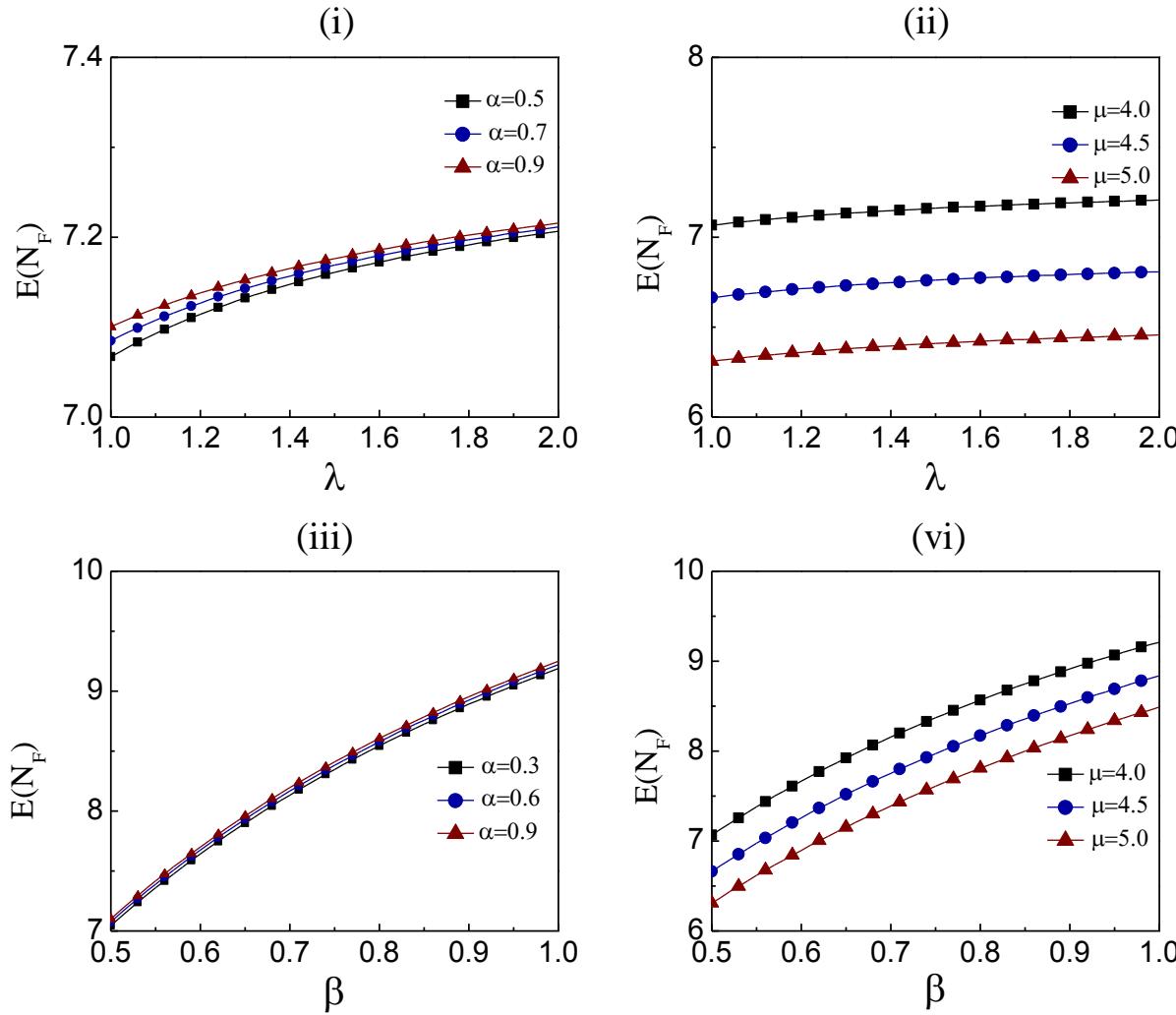


Fig. 6.3: Expected number of failed machine  $E(N_F)$  by varying  
 (i)  $(\lambda, \alpha)$ , (ii)  $(\lambda, \mu)$ , (iii)  $(\beta, \alpha)$  and (iv)  $(\beta, \mu)$ .

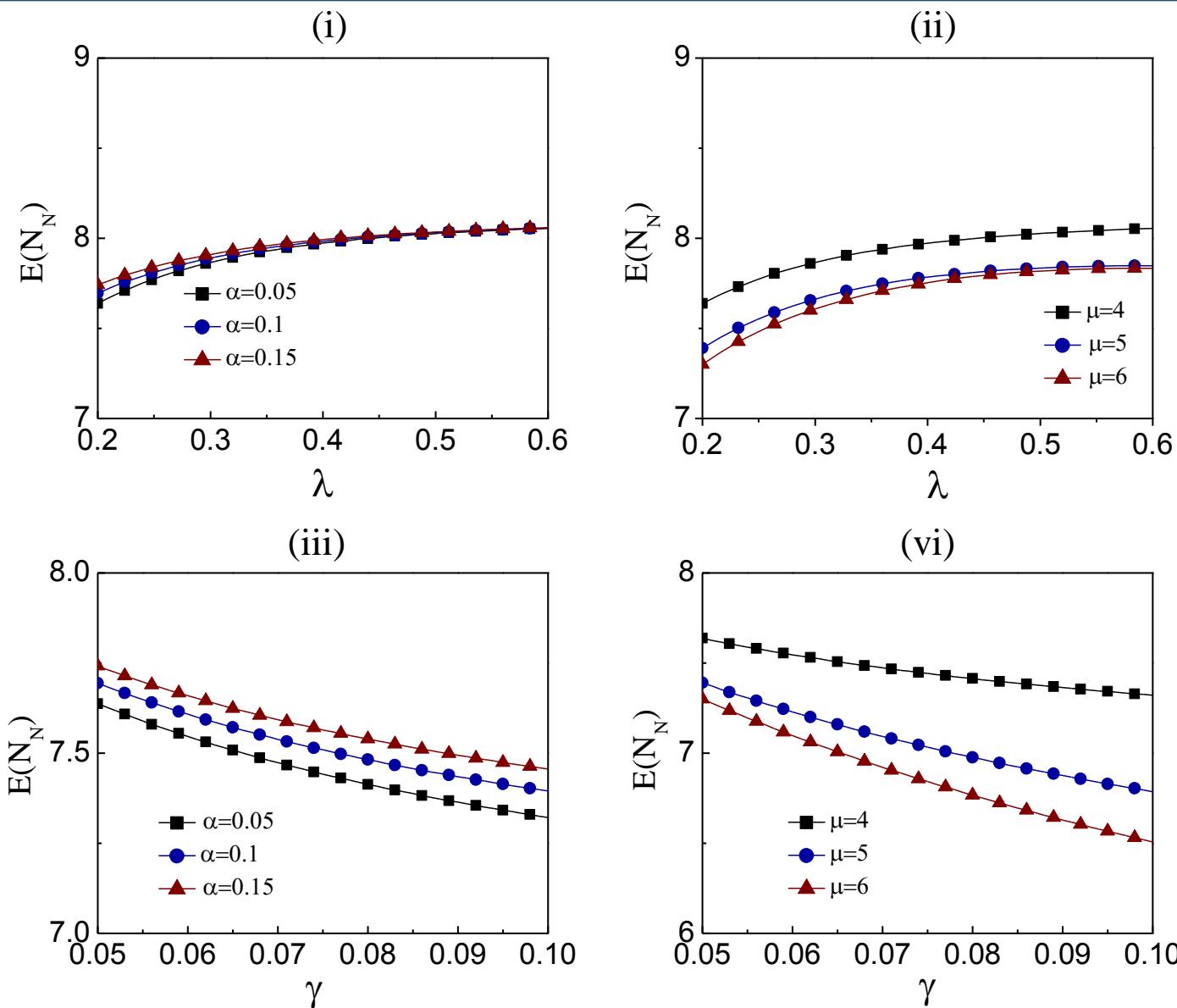


Fig. 6.4: Expected number of failed machine  $E(N_N)$  by varying  
 (i)  $(\lambda, \alpha)$ , (ii)  $(\lambda, \mu)$ , (iii)  $(\gamma, \alpha)$  and (iv)  $(\gamma, \mu)$ .

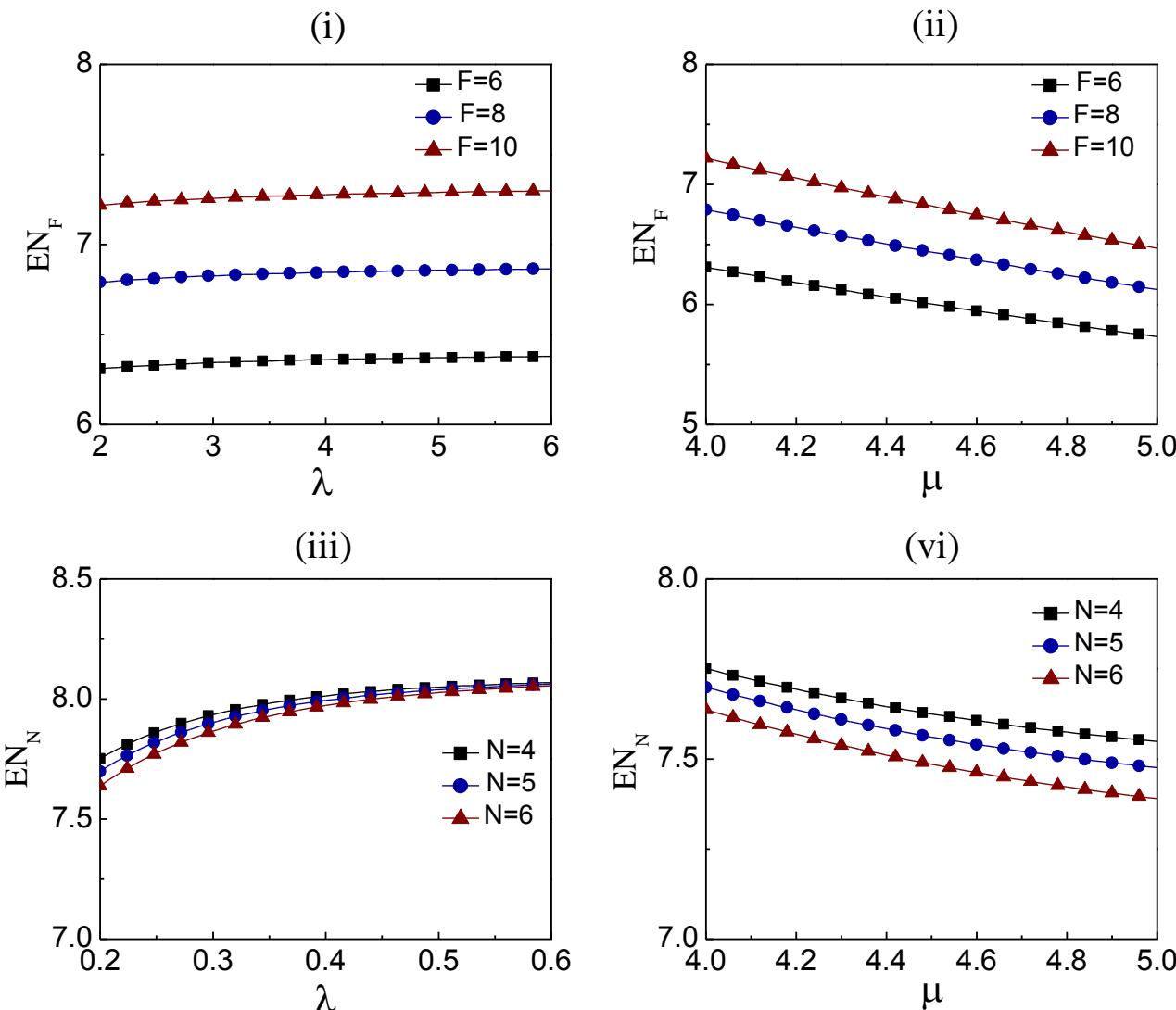


Fig. 6.5: Expected number of failed machine (i)  $E(N_F)$  by varying  $(\lambda, F)$ ,  
(ii)  $E(N_F)$  by varying  $(\mu, F)$ , (iii)  $E(N_F)$  by varying  $(\lambda, N)$ , (iv)  $E(N_F)$  by varying  $(\mu, N)$ .

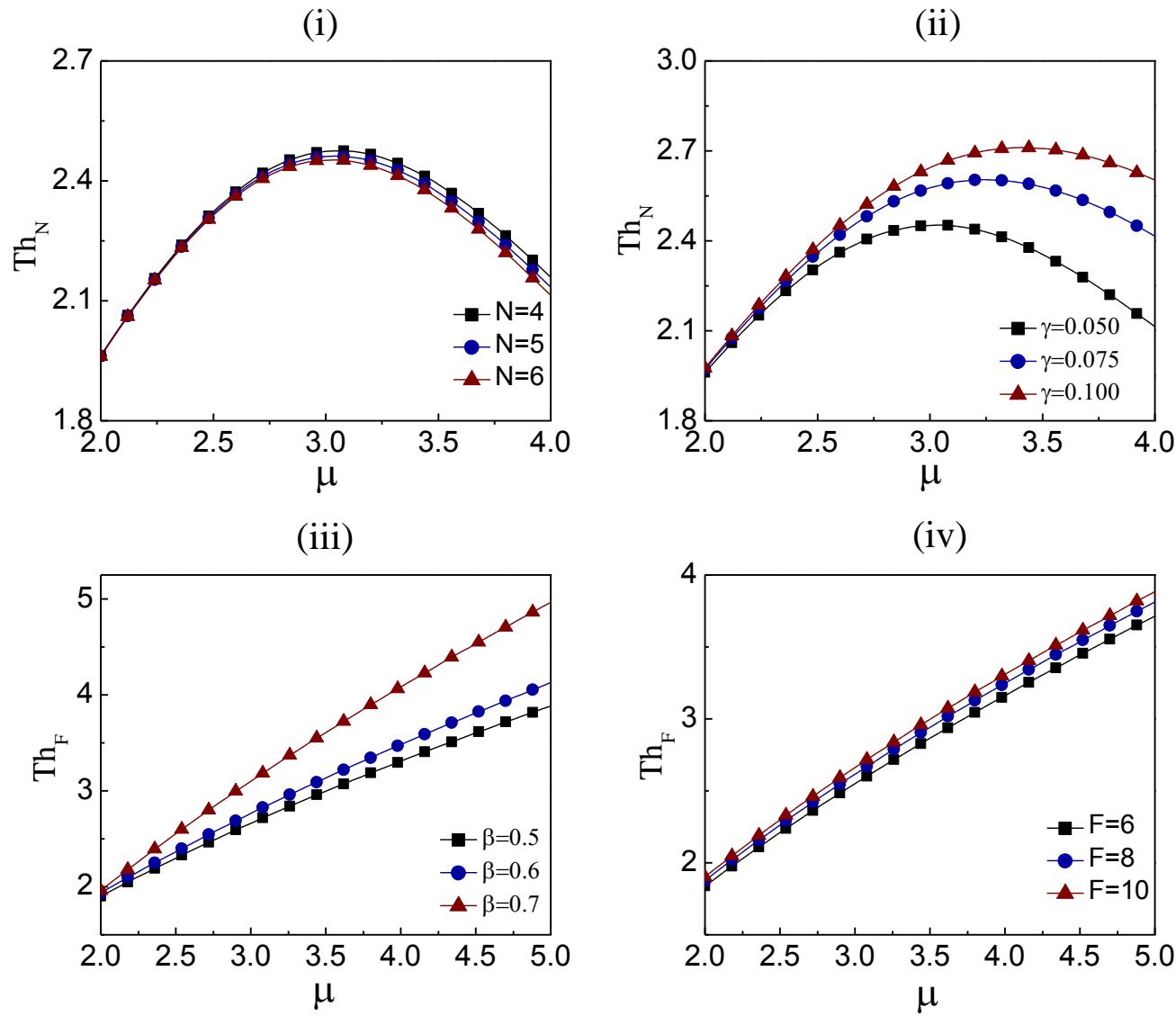


Fig. 6.6: Throughput of the system (i)  $\text{Th}_N$  by varying  $(\mu, N)$ , (ii)  $\text{Th}_N$  by varying  $(\mu, \gamma)$ , (iii)  $\text{Th}_F$  by varying  $(\mu, \beta)$ , (iv)  $\text{Th}_F$  by varying  $(\mu, F)$ .

## Conclusions

- ❖ The average number of failed machines in case of both policies seems to increase with the increase in the failure rate of operating machines, failure rate of standbys machines and setup rate.
- ❖ The throughput of the system for N-policy model initially increases then decreases. But, for F-policy model, throughput increases sharply; it is due to the fact that the server is always available for the service in case of F-policy model.